

タイトル	How Long Should the Public Sector Provide Public Education through Economic Growth?
著者	Hemmi, Noriyoshi
引用	季刊北海学園大学経済論集, 67(2): 53-58
発行日	2019-09-30

# How Long Should the Public Sector Provide Public Education through Economic Growth?

Noriyoshi Hemmi\*

## Abstract

We have investigated how educational ability-screening affects the demand for education. If ability-screening is delayed, high-ability individuals might be undereducated, or low-ability individuals might be overeducated. When society undereducates high-ability individuals, the public sector can solve the problem by adjusting the length of public education. In addition, setting a shorter public education term in the runup to a private education, improves efficiency through economic growth. When society overeducates low-ability individuals, setting a longer public education term in the runup to a private education, is a socially optimal.

## 1 Introduction

Although knowledge and skills can be acquired through education, some abilities are innate. Information about individual ability is not always available, even to the individuals themselves, especially at an early stage of education. This uncertainty sheds light on the role of education in screening individual ability. For example, ability-screening in education may be beneficial for more efficient and effective human resource allocation (Arrow, 1973). At the same time, it may exacerbate inequality in future income. If so, tax-financed public education remains controversial (Stiglitz, 1975). In the existing literature, economic justifications for education subsidies rely mostly on the positive externalities of higher education (Wigger, 2001), incomplete markets for educational loans (Carneiro and Heckman, 2002), and missed opportunities to insure against educational risks (Wigger and von Weizsäcker, 2001). In more recent works, Oshio and Yasuoka (2009), in which an education has no externalities, discuss the demand for education in several educational systems and the shape of an optimal education system, opining that publicly subsidized education is efficient because it limits educational investment.

This paper is based on Oshio and Yasuoka (2009) and Hemmi (2018). We modify one point, the decision about whether or not to stay in school, which is based on the net benefit of dropping out.

---

\* Faculty of Economics HOKKAI GAKUEN UNIVERSITY 4-1-40, Asahi-Machi, Toyohira-Ku, Sapporo, 062-8605, Japan. e-mail: nhemmi@hgu.jp

This modification simplifies Oshio and Yasuoka's explanation and makes it possible to analyze not only overinvestment but also underinvestment in education. For both underinvestment and overinvestment, this paper shows that how long the public sector should set the length of public education in the runup to a private education. In addition, we show that for underinvestment and overinvestment, a shorter and a longer public education term improves efficiency through economic growth, respectively.

## 2 Model

We divide students into high-ability ( $i=H$ ) and low-ability ( $i=L$ ), who are represented as  $p$  and  $1-p$  ( $0 < p < 1$ ), respectively, for the total population. After receiving education of length  $x$ , each type of individual obtains an educational output worth  $a_i x^\varepsilon$ .  $a_i$  ( $i=H, L$ ) is a positive parameter of an individual's innate ability, and we assume that  $a_H > a_L$  and  $0 < \varepsilon \leq 1$ . After receiving education of length  $x$ , the net benefit received from this education,  $W_i(x)$  ( $i=H, L$ ) is expressed as

$$W_i(x) = \int_x^\infty a_i x^\varepsilon e^{-rs} ds - \int_0^x c e^{-rs} ds$$

where  $c$  is the unit cost of education and  $r$  is the discount rate. If individuals are fully informed about their abilities in advance, they choose  $x_i$  ( $i=H, L$ ) to maximize  $W_i(x)$ . From  $a_H > a_L$ , we have  $x_H > x_L$ . Because  $\varepsilon$  is an elasticity of educational output, and  $r$  is the discount rate, it is reasonable to assume that  $\varepsilon > r$ . Under this assumption, we can normalize  $x_H$  as 1, without loss of generality.

Since we do not consider the positive externalities of education, the net social benefit from education is maximized when two types of individuals receive education of length  $x_H$  and  $x_L$ , respectively.

## 3 Conditions for Staying in School

We assume that the government establishes an education system with a total length of 1. In addition, we assume that individuals do not know their abilities before receiving an education and an individual who drops out before completing his education is always treated as low-ability. After receiving an education of length  $x$ , individuals conjecture that they are high-ability with probability  $\theta_i(x)$ , ( $i=H, L$ ,  $\theta_H(x) > 0$ ,  $\theta_L(x) < 0$ ,  $\theta_H(x) > \theta_L(x)$ ). Thus, the condition for staying in school at  $x$  is

$$W_L(x) < \theta_i(x) W_H(1) + (1 - \theta_i(x)) W_L(1), i=H, L$$

From the above inequality, we derive the following formula:

$$\theta_i(x) > \frac{W_L(x) - W_L(1)}{W_H(1) - W_L(1)}, i=H, L \quad (1)$$

In a non-subsidized education system (private system), the graph of the right-hand side of

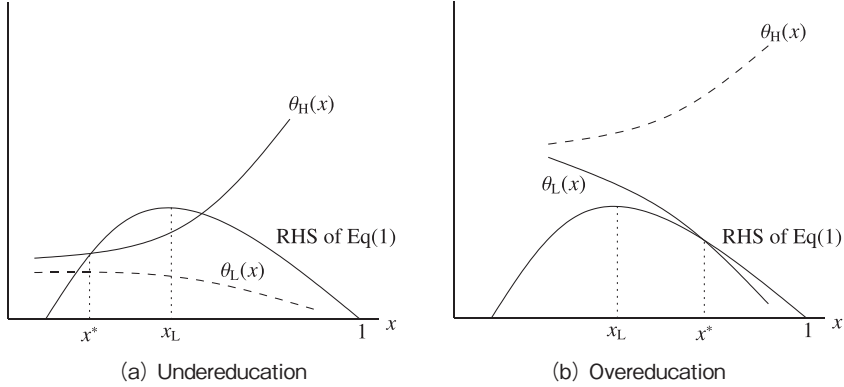


Figure 1: Timing of Dropouts in Private System: (a) High- and low-ability individuals drop out at  $x_L$ . (b) Low-ability individuals drop out at  $x^*$ , and high-ability individuals complete their education.

Equation (1) has a form similar to the graph of  $W_L(x)$ , which peaks at  $x_L$ . Figure 1-(a) shows the case, where both individuals do not complete their education but receive education  $x_L$ . This means that high-ability individuals receive a less than socially optimal education. Figure 1-(b) shows the case, where high-ability individuals complete their education, and low-ability individuals do not but still receive a more than socially optimal education.

#### 4 Length of Public Education

In a mixed education, the public sector provides a partial public education of length  $\lambda$  ( $0 \leq \lambda < 1$ ), through which all individuals can receive tax-financed education between 0 and  $\lambda$ . After completing public education, they can receive additional private education if they wish and drop out at any time. Thus, the net benefits received by high- and low-ability individuals who work are expressed as

$$W_H(1) = (1-t) \int_1^\infty a_H e^{-rs} ds - \int_\lambda^1 c e^{-rs} ds \quad (2)$$

$$W_L(x) = (1-t) \int_x^\infty a_L x^\varepsilon e^{-rs} ds - \int_\lambda^x c e^{-rs} ds \quad (3)$$

Public education costs are fully financed by a wage-proportional tax  $t$ , which is solved by

$$\int_0^\lambda c e^{-rs} ds = t \times \left[ p \int_1^\infty a_H e^{-rs} ds + (1-p) \int_x^\infty a_L x^\varepsilon e^{-rs} ds \right] \quad (4)$$

From Equations (2), (3), and (4), we have  $\partial W_H(1)/\partial \lambda < 0$ ,  $\partial W_L(x)/\partial \lambda > 0$ , and  $\partial W_L(1)/\partial \lambda > 0$ . Therefore, an increase in public education  $\lambda$  causes the graph of the right-hand side of Equation (1) to shift upwards.

Fig 2 shows how long the public sector should set the length of public education to address the undereducation of high-ability individuals or to minimize the overeducation.

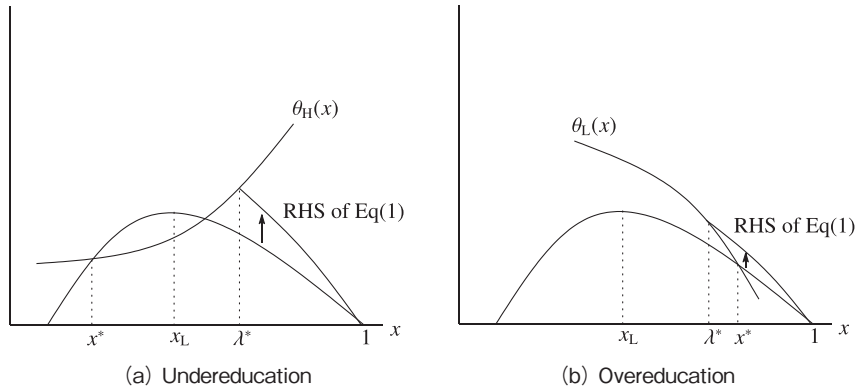


Figure 2: Length of Public Education in Mixed System: (a) Public education  $\lambda^*$  leads high-ability individuals to complete their education. (b) Public education  $\lambda^*$  leads low-ability individuals to drop out at the minimum overeducation.

## 5 Policy Implications

### 5.1 Net Social Benefit from Education

When society undereducates high-ability individuals in the private education system, the public sector can set the length of public education to address this issue. However, public education also overeducates low-ability students. While the former improves efficiency, the latter reduces it. Whether or not a mixed system is socially optimal depends on their relative level. A mixed system likely will improve efficiency under certain conditions: the difference of innate ability  $a_H - a_L$  is larger, the proportion of high-ability  $p$  is higher, and the socially optimal length of education for low-ability  $x_L$  also is longer.

When society faces overeducation of low-ability individuals in the private education system, the public sector can set the length of public education, which causes low-ability individuals to drop out at the minimum overeducation. Also, when the public sector sets the length of public education at  $\lambda = 1$ , this education system becomes fully public. But since there is no externality, a fully public system is not a socially optimal. Thus, in this case, a mixed system can be more efficient than other systems.

### 5.2 Economic Growth and Length of Public Education

Life cycle skill formation is a dynamic process, in which early inputs strongly affect the productivity of later inputs (Heckman 2006). Higher income from economic growth makes it possible to invest in pre-school training. Thus, economic growth leads to a higher proportion of high-ability students, which makes  $\theta(x)$  higher. Also, it is reasonable to assume that  $a_H$ ,  $a_L$  and  $c$  increase at the same rate.

How long should the public sector set the length of public education through economic growth? Fig 3 shows that when society undereducates high-ability individuals in the private education system, a shorter public education term leads high-ability individuals to complete their education.

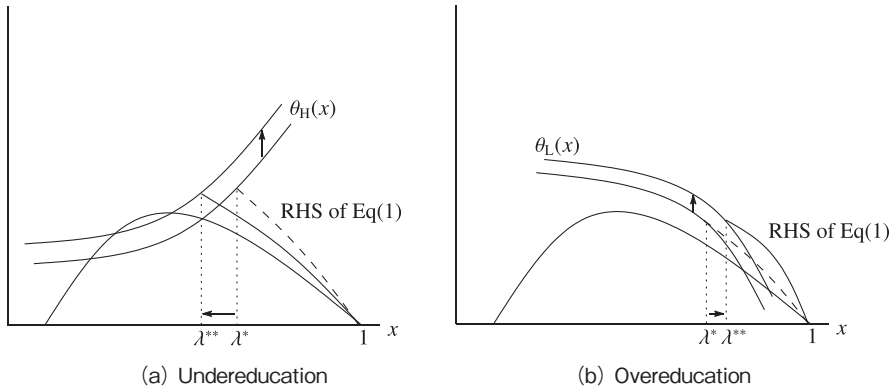


Figure 3: Length of Public Education through economic growth: (a) A shorter public education  $\lambda^{**}$  improves efficiency. (b) A longer public education  $\lambda^{**}$  improves efficiency.

When society overeducates low-ability individuals, the public sector should set a longer public education term to lead low-ability individuals to drop out at the minimum overeducation.

## 6 Conclusion

We have investigated how educational ability-screening affects the demand for education. If ability-screening is delayed, high-ability individuals might be undereducated, or conversely, low-ability individuals might be overeducated. In both cases, the public sector can solve the problem by adjusting the length of public education.

When society undereducates high-ability individuals, this intervention also diminishes education efficiency by overeducating low-ability individuals. However, through economic growth, a shorter public education is sufficient to lead high-ability individuals to complete their education.

When society overeducates low-ability individuals, the public sector can set the length of public education, which causes low-ability individuals to drop out at the minimum overeducation. However, through economic growth, the public sector have to set a longer public education term to address this issue.

## ⟨References⟩

- Arrow, K. J. (1973): ‘Higher education as a filter’, *Journal of Public Economics*, 2, pp.193-216.
- Carneiro, P., Heckman, J. (2002): ‘The Evidence on Credit Constraints in Post Secondary Schooling’, *Economic Journal*, 112, pp.705-734.
- Heckman, J. J. (2006): ‘Skill formation and the economics of investing in disadvantaged children’, *Science*, 30, pp. 1900-1902.
- Hemmi, N. (2018): ‘Fully Public Education and Low-Ability Individuals’ Welfare’, *Hokkai-Gakuen University, the journal of economics*, 66, pp.55-61.
- Oshio, T., Yasuoka, M. (2009): ‘How long should we stay in education if ability is screened?’, *Metroeconomica*, 60,

pp.409-431.

Stiglitz, J. E. (1975): 'The theory of "screening", education, and the distribution of income', *American Economic Review*, 65, pp.283-300.

Wigger, B. U. (2001): 'Pareto-efficient Intergenerational Transfers', *Oxford Economic Papers*, 53, pp.260-280.

Wigger, B. U., von Weizsäcker, R. K. (2001): 'Risk, Resources and Education — Public versus Private Financing of Higher Education', *IMF Staff Papers*, 48, pp.547-560.